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More Efficient Solution of Equations Using New Technological Solutions and Knowledge Bases

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Abstract

The main goal of this paper is to research the problems related to the rapid acquisition of necessary knowledge in solving mathematical equations with the application of appropriate knowledge bases and semantic web. The research carried out, as well as the insight into the domestic and available foreign literature, show that this paper deals with a new and unique model of acquiring mathematical knowledge with the use of new technological solutions. Knowledge bases and semantic web are something completely new in the practical application of new technologies, and in this paper they are used and applied for the first time for updating knowledge and more efficient and easy access to it. Here we create a different view on the problem of acquiring the necessary mathematical knowledge in solving equations. In this paper we start from the number as one of the solutions of the equation, over the linear equations, and some other types of equations, to the differential equations of higher order. We updated part of the knowledge with the use of Protégé, an open source platform, which can provide fast, easy and efficient access to up-to-date knowledge.

Keywords: equations, knowledge bases, knowledge presentation, semantic web, Protégé editor

Introduction

Paying more attention to quick access to the necessary knowledge is very important for fastsolving certain, and especially mathematical problems. It is not only significant to have basic knowledge of mathematics and solving mathematical problems, as well as effective and fast approach to the necessary mathematical knowledge, but it is necessary to be persistant and dedicated to find the correct solution [1].

The modern world must be able to adapt quickly to new technological solutions and adapt to an environment that is changing rapidly, and therefore must use new technological solutions, in particular in quickly reaching the necessary knowledge. Therefore, we can say that "the use of the knowledge bases implies the acceptance of new technological ideas for quicker access to the necessary knowledge [2].“

In order to use a new methodology and a new approach to observing mathematical problems and solving equations using new technological solutions and knowledge bases, we will introduce some more concepts that are necessary. These are primarily semantic web and knowledge bases [3].

The phenomenon of Semantic Web was introduced by Tim Berners Lee (2001) as a clear structure to the content of the website. It was created due to a need for more efficient finding of certain information and knowledge. It is based on the idea that information on the web should become machine-readable.Interconnected data (information) that has the specified structure and meaning should be used instead of documents linked by hyperlinks.

In order for the Semantic Web idea to work, computers should have access to information collections. It must provide rules for data reasoning, and enable the presentation of data and information.

To update only a part of the mathematical knowledge necessary for solving equations, we will use the *Protégé* editor, an open source platform that provides a rich set of modeling structures and activities that support the creation, visualization, and manipulation of data and information that are represented in different formats. With the use of *Protégé* editor, we updated some of the basic of mathematical knowledge - Figure 1.

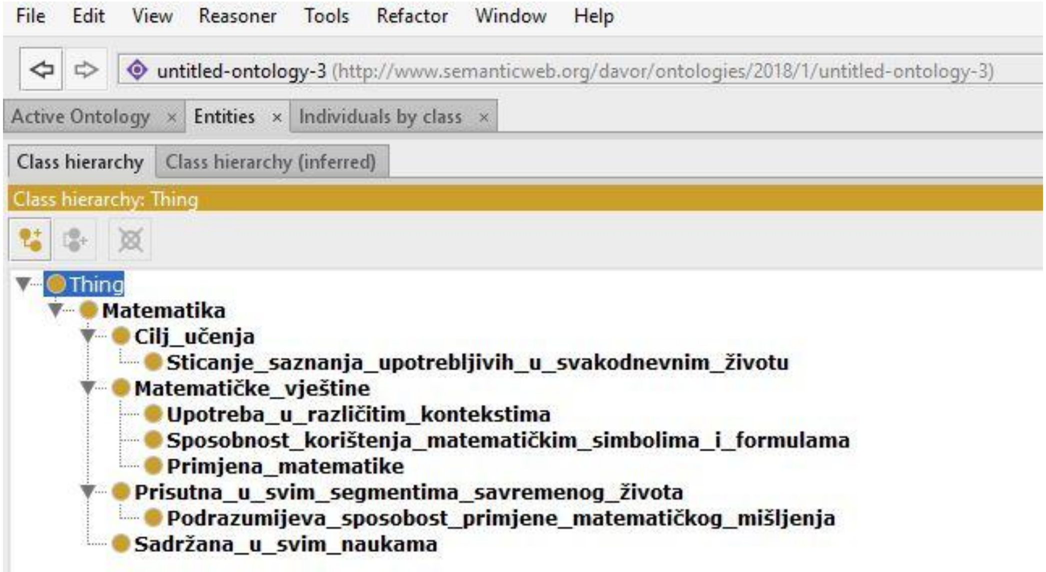

Figure 1. Mathematics

1. The concept of the number

Here we give basic concepts and knowledge of the concept of the number, since the solutions to the equations are very often certain numbers.

Number is one of the basic concepts on which today's mathematics is based, and it was created due to the need for counting subjects. It has been perfected in proportion to the development of mathematical knowledge. If development is monitored through history, it can be seen that ancient scientists have found that a series of natural numbers is infinite (3rd century BC).

Since ancient times, people have had a need for certain evaluations of how large a group of items is, so the problem was solved by comparing it with fingers, sticks, a pile of stones and other objects. From such a comparison much later an abstract concept of the number developed, unrelated to the kind of things we count, as we have it today. In those early days, at an early stage of development only very small numbers could be perceived by people, and the rest was expressed by much or many.

When people adopted the concept of the number, they were not immediately able to use them in mathematical operations, however simple they were, not even with the smallest numbers [4].

Today, we express numbers by the degree of number 10. This is probably because our ancestors in the early development used fingers as a means of counting and as the first "calculator".

Numbers are everywhere around us: when we look at the price of goods in the store or pay invoices and receipts at a cashier or in a bank, when we receive a salary, when we enter someone's phone number, we look at the clock, or write down a significant date. We have been using it so much in writing that we are not thinking why we write the numbers exactly like that, and whether people ever wrote them differently.

Some authors define numbers as changing types of words used for expressing of how many things there are or which position is something in a list. Depending on what they are expressing, numbers are divided into cardinal and ordinal numbers.

The cardinal numbers are the numbers that express how many things there are. Based on what they express the numbers are divided into several subcategories [5].

Natural numbers are infinitely many and it was not possible to invent a separate name for each of them. Therefore, words were invented only for a small quantity of numbers, and the other numbers are expressed by combinations of these words [6].

The numbers we use today are called the Arabic numerals because we took them over from the Arabs in the Middle Ages, and they took them over from the Indians.

The first release of natural numbers were rational numbers, fractions, created with the need to measure some size, compared with some other size - the etalon. All subsequent extensions of the concept of number did not arise due to the needs of computation and measurement, but were the consequence of the development of science.

Types of numbers are:

Natural numbers: - are all numbers larger than zero (0). A set of natural numbers is most commonly referred to as N = {1, 2, 3, 4, ...}.

For each number $n \in N$ there is number $n + 1 \in N$. Numbers n and n +1 are consecutive numbers.

A natural number whose only divisors are itself and number 1 is called a prime number, for example: 2,3,5,7,11, ... As mutually prime numbers we call two natural numbers if their only common factor is number 1.

A natural number is even if at least one of its prime factors is number 2. If this is not the case, the number is odd. The even numbers are denoted with 2k, and odd with 2k + 1 or 2k - 1, where $k \in N$.

A set of natural numbers is closed in relation to the addition and multiplication operations, i.e. the result of adding and multiplying two natural numbers is always natural number.

Integers - make the set of all natural numbers, zero, and all negative numbers and must not have a decimal extension. We denote them with Z = {0,1, -1,2, -2, ...}.

Subtraction in the set of integers is an operation which is defined as:

$$\forall\ a,b,c \in Z\ ,\ a - b = c, \Leftrightarrow a = b + c\ .$$

The set Z is a subset of the set N and it retains all the rules that we defined in the set N, adding some new ones that are valid only in it. We will also use this principle in defining the next expending of sets of numbers.

Rational numbers are: integer numbers + fractions. These are the numbers that can be written as the ratio of integers a/b where b is not zero. They can be written in an infinite number of ways, for example: ½, 2/3, 3/5, 7/6 {valid 1/2 = 2/4 = 3/6 = ...}

Irrational number: - is the real number that is not a rational number, i.e. it can not be written as a fraction of two integers, thus it cannot be written as a/b. (These can be the roots of the number).

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \log 2, \pi, e ,...., .$$

All rational and irrational numbers form a set of real numbers, i.e. R = Q U I so we can say that real numbers are: rational + irrational. Let's denote them with R, and for them the following is valid:

$$(\alpha > \beta) \Leftrightarrow (\beta < \alpha)\ ;$$

$$(\alpha > \beta) \Leftrightarrow (\alpha + \chi > \beta + \chi)\ ;$$

$$(\alpha > \beta\ ,\ \chi > 0) \Rightarrow (\alpha\chi > \beta\chi)\ ,\ (\alpha > \beta\ ,\ \chi < 0) \Rightarrow (\alpha\chi < \beta\chi)\ .$$

The absolute value of a number is always a non-negative number, i.e. $|\ a\ | \geq 0$

Complex numbers are: real + imaginary. These are all the numbers in the form a + bi

Not all the square equations have solutions in a set of real numbers, for example, the equation $x^2 + 1 = 0$. In order to solve this very simple equation we need to expand the set of real numbers. This way we come to the set of ***complex*** numbers.

Imaginary unit is by definition $i = \sqrt{-1}$

The solution of this equation now becomes $x = \pm\sqrt{-1} = \pm i$.

A set of all ordered pairs of real numbers (x, y) in which z = x + iy, i.e.

$z = (x, y)$ is called a set of complex numbers C, where$i = \sqrt{-1}$

The real part of the complex number is Re $(z) = x$, imaginary partIm$(z) = y$.

Two complex numbers $z_1 = x_1 + iy_1$and$z_2 = x_2 + iy_2$are equal if their realistic parts are the same for themselves and imaginary for themselves; x1 = x2 andy1 = y2.

Using the Protégé editor, we will update only a part of the knowledge related to the term number and present it in Figure 2.

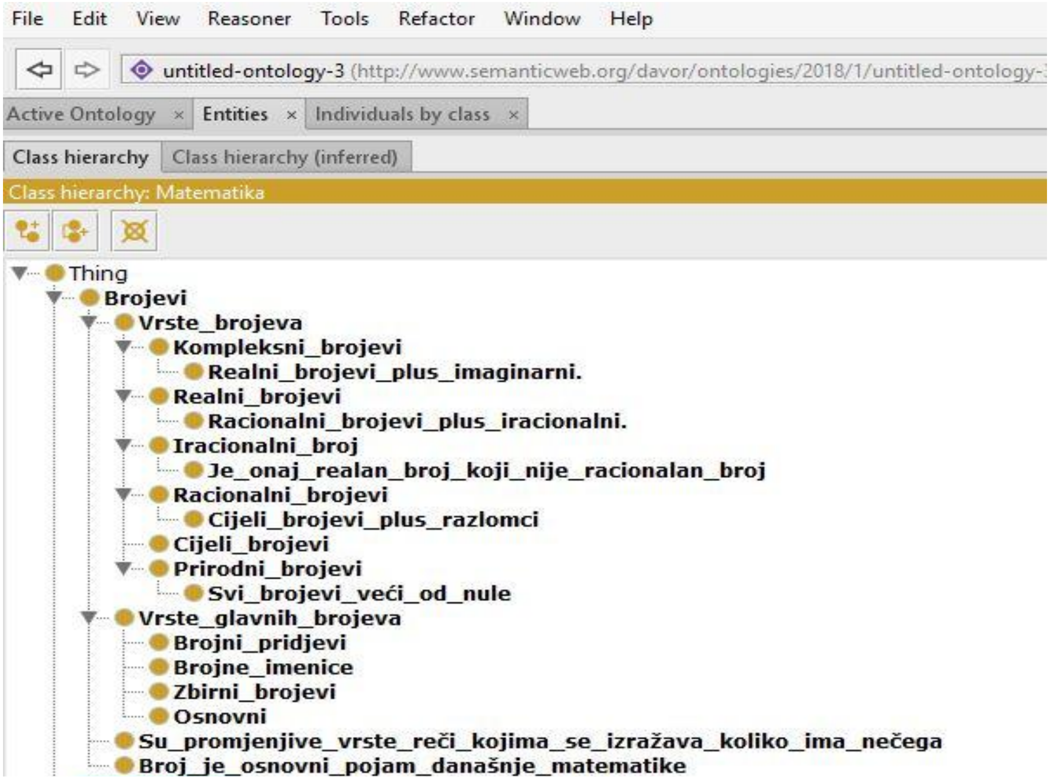

Figure 2. Numbers

Properties of opposite numbers

For every two real numbers a and b:–(-a) = a(-a)b = -(ab) = a(-b) = -ab(-a)(-b) = ab

(-1)a = -a$\frac{-a}{b}$ = -$\frac{a}{b}$ = $\frac{a}{-b}$, b ≠ 0 $\frac{-a}{-b}$ = -$\frac{-a}{b}$ = - −$\frac{a}{-b}$ = $\frac{a}{b}$, b ≠ 0

Properties of zero

For every two real numbers a and b:a * 0 = 0ab = 0 only and only if a = 0 or b = 0

Complex numbers

The algebraic representation of the complex number is: z = a + bi or z =Re(z)+Im(z)i, where a is a real part, and b is the imaginary part of the complex number.

Basic computational/mathematical operations with complex numbers

Addition - (a+bi)+(c+di) = a+bi+c+di = (a+c)+(b+d)i

Subtraction - (a+bi)-(c+di) = a+bi-c-di = (a-c)+(b-d)i

Multiplication - (a+bi)(c+di) = ac+adi+bci+bdi2 = (ac-bd)+(ad+bc)i

2. Equations

Equation is a term in mathematics that expresses the connection between known and unknown sizes by means of the equality sign that equates the left and the right side of the equation. It is possible to distinguish a mathematical identity, where only the equality of the left and the right is determined, from the equation where it is searched for the value of an unknown item so that it satisfies the set equation. Unknown items, unknowns, are usually denotedas: x, y, z, or any other mark.

Equations are solved by some of already standard procedures (methods), with equations varying according to the properties and method of solving.

Solving the equation consists of determining which values of the variables make the equality true. Variables are also called the unknowns, and values of unknowns that satisfy the equation are called equations solutions.

Linear equation

This is the simplest equation in form of: ax + b = 0

The solution of the linear equation by an unknown item x is also the null point of the linear function.

Systems of two equations with two unknowns

When we talk about the system of two linear equations with two unknowns x and y, we mean the so-called "simple" system, which we can always achieve with equivalent transformations. The general form of a system of two equations with two unknowns x and y is:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

where a1, a2, b1, b2, c1, c2 are given real numbers.

Systems of three equations with three unknowns

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

As an example of three equations with three unknowns we can take a system:

$$x - y - z = 13$$

$$2x + 5y - 3z = -13$$

$$4x - y - 6z = -4$$

Where solutions are x = 6, y = - 2, z = 5or (6, -2, 5)

Due to the length of the work, we will not specify how to solve the linear equation, how to solve a system of two linear equations with two or three unknowns.

Systems of linear equations with n unknowns

A system of n linear equations with n unknowns (x1, x2, ...,xn) has a general form:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1,$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2,$$

....

....

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n.$$

We will mention only Gaussian method for solving the equation system consisting in the successive elimination of unknowns from the system and transformation into a triangular or trapezoid equivalent system from which a solution is obtained or it is found that the system has no solution[7].

Suppose that the coefficient $a_{11} \neq 0$. To exclude the unknown x1 from all system equations except from the first one.

In order to do this it is necessary to multiply the first equation with: $-\frac{a21}{a11}$ and add it to the second equation, then multiply the first equation with: $-\frac{a31}{a11}$ and add it to the third equation, etc. In this way, instead of the starting system, an equivalent system is obtained that needs to be further simplified until a definitive system solution is achieved[8].

Operations that produce equivalent systems

A system of linear equations transforms into its equivalent system if:

(A) Two equations replace places.
(B) The system equation is multiplied with a constant different from zero.
(C) One equation, multiplied by a constant, we join to another equation.

It can be shown that each linear equations system must have exactly one solution, have no solution, or have infinitely many solutions, regardless of the number of equations or variables in the system. To describe these solutions, the terms used are a unique solution, consistent, non- consistent, dependent and independent, just like in the case of two variables. A part of the knowledge about elementary equations was updated and presented in Figure 3

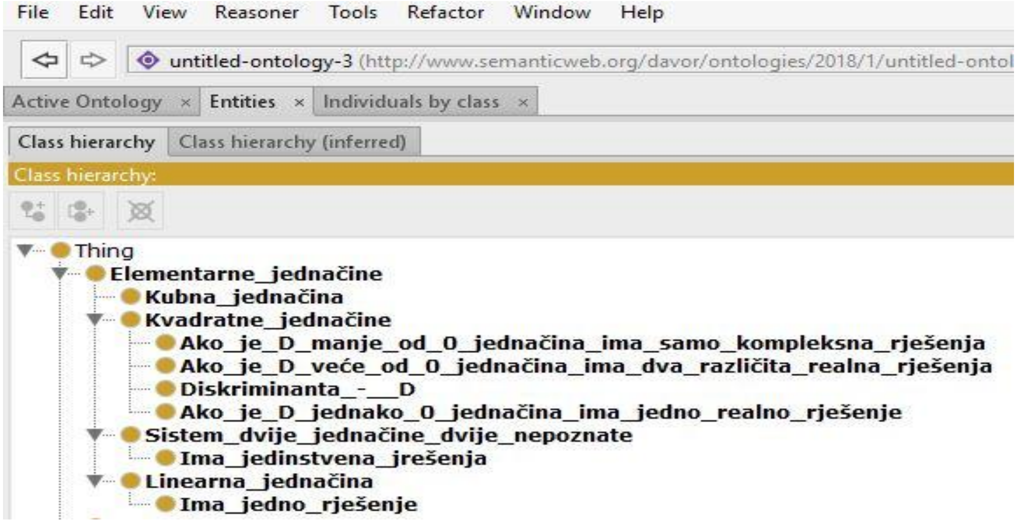

Figure 3. Elementary equations

2. 1. Square equations

The equation of form $\mathbf{ax^2 + bx + c = 0}$, where x is unknown and where a, b, and c are real numbers, where $a \neq 0$ is called a square equation.

Every square equation in general has two solutions and these are presented in the form of:

$$\mathbf{X}_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

In order to solve the square equation we reduce it to the general form, and its solutions are calculated according to the above formula[9].

The term under (inside) the root is called a discriminant ($D = b^2 - 4ac$). The discriminant can be used to analyze the nature of the solution of a square equation without solving it, and it can be greater than zero ($D> 0$), less than ($D <0$) and equal to zero ($D = 0$).

If $b^2 - 4ac > 0$, equation has two different real solutions.

If $b^2 - 4ac < 0$, equation has only complex solutions.

If $b^2 - 4ac = 0$, equation has only one real solution, double root.

Triple equations

Triple equation has the general form:

$$ax^3 + bx^2 + cx + d =0$$

The solution of the triple equation, depending on the values of a, b, c and d, can have one real and two conjugated complex solutions, three different real solutions, or two equal real solutions, and the third, different, also realistic solution

With the use of *Protégé* editor, the open source platform, we will update only part of the knowledge related to square equations Figure 4.

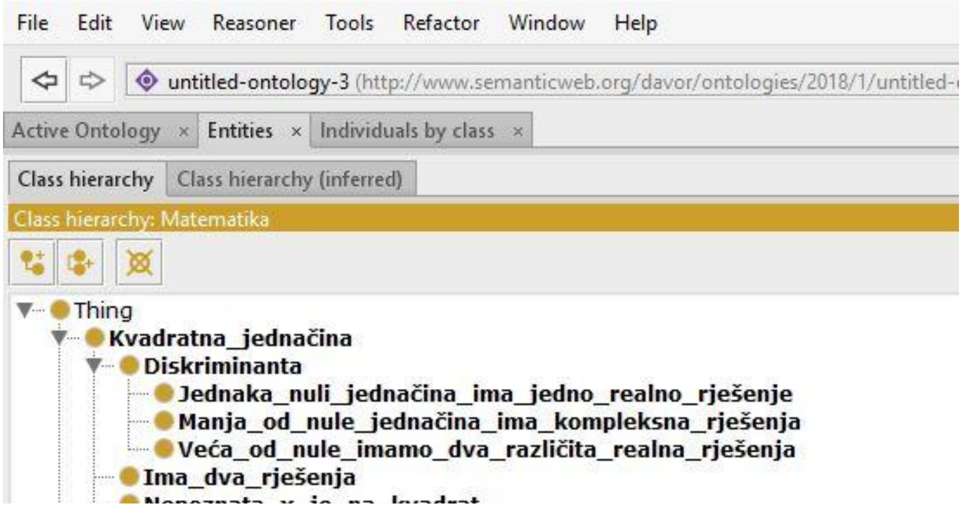

Figure 4. Square equation

Polynomial equation

Polynomial equation has the general form:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots\, a_2 x^2 + a_1 x + a_0 = 0$$

It is defined for all values of unknown x, where n is positive integer and a_0, a_1, a_2, ..., a_n and are equation coefficients. The equation has n solutions, where the solutions of the equation are in the whole complex plane.

Irrationalequation

It is the equation where unknown item appears inside the root, for example:

$$\sqrt{3x+2} + 7 = \sqrt{4x-7}$$

Exponential equation

It is the equation where unknown item appears in the exponent, and as an example we can observe the following exponential equations:

$$5^{2x} - 90 = 0 \text{or} 10^{2x+1} = 1000^{5x-2}$$

Logarithmic equation

Is the equation where the unknown item is contained within the logarithm or makes the basis of logarithm:

$$\text{Log}3x - 1/3\text{Log}x = 9$$

Trigonometric equation

is it the equation where the unknown item is an argument of the trigonometric function, such as:

$$\sin^2 x + \sin x = 0 \text{or} \text{ctg}2x = 3\text{ctg}x$$

We updated only some knowledge of equations and presented it in Figure 5.

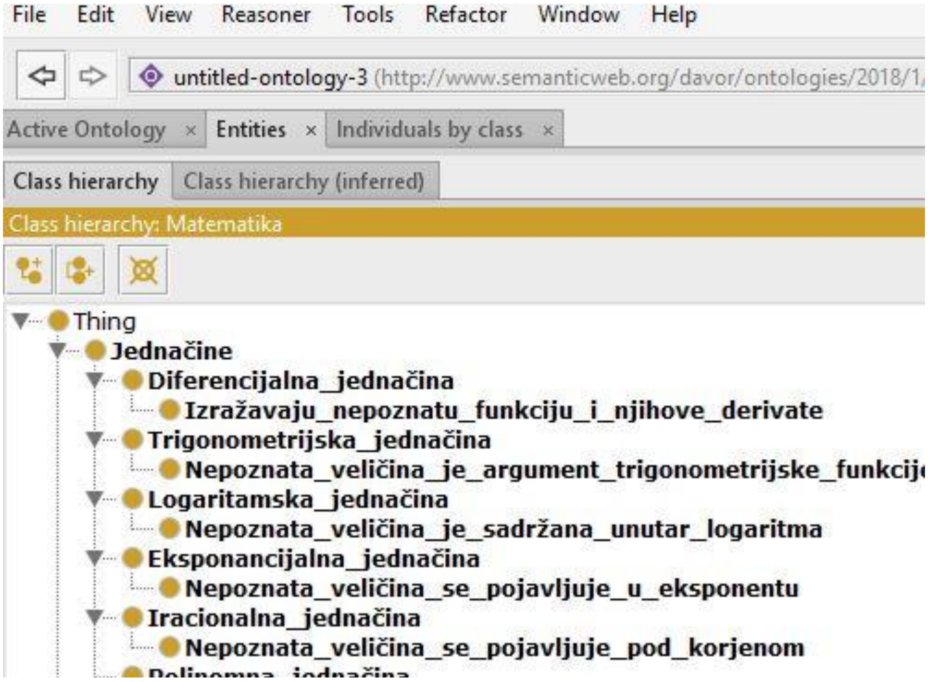

Figure 5. Equations

3. Differential equation

Differential equation is any equation in which an independent variable (x) appears, unknown functionof that variable (f(x).) andderivatives or differentials of an unknown function. By definition, the *order* of the differential equation is the highest order of the derivative in that equation. The general form of the differential equation of the *n*-th order is: $F(x, y, y', y'',\ldots, y^{(n)}) = 0$

This is theequationwhich expresses the relation between an independent variable, an unknown function, and its derivatives: $F(x, y, y', y,\ldots, y^{(n)}) = 0$. *The highest order of a derivative in this equation is called the order of differential equation.* For *example, $y'' + ky^3 = 0$ is a differential equation of the second order*. The simplest differential equation is of the first order, in an explicit form it is the equation of the form $y' = f(x)$.

These are *solved* by finding an expression for a function that does not include derivatives. Differential equations are used to model processes that include the variation rates of variables, and can be applied in fields such as physics, chemistry, biology, and economics[10].

If all the unknowns of the function which are part of the differential equation depend only on one independent variable, and therefore the equation does not contain partial derivatives, this equation is called the ordinary differential equation.

If there are unknown functions in the equation that depend on more independent variables, so the partial derivatives of unknown functions appear in the equation, this equation is called a partial differential equation.

The equation form $y'(x)=2x$ is differential equation of the first order.

The equation form $y''(x)=2x$ is differential equation of the second order.

The equation form $y'''(x)=2x$ is differential equation of the third order, etc.

The solution of the differential equation is such a function g(x) which by replacing it in this equation gives the identity: $f(x, g(x), g'(x), g''(x), \ldots, g^{(n)}(x))=0$

3. 1. Homogeneous equation

Function$f(x, y)$depends on the relation $\frac{y}{x}$, $f(x, y) = f(\frac{y}{x})$is called **homogeneous equation**. Differential equation y′ = f (x, y) in which the function f(x, y) is homogeneous $y' = f(\frac{y}{x})$represents homogeneous differential equation.

This type of equation is reduced by introducing a substitution to the equation: $u = \frac{y}{x}$ or y = ux, and by differentiation of the left and right sides result is: y '= u'x + u, so the starting equation can be transformed into the equation $u'x + u = f(u)$

3. 2. Linear differential equation of the first order

Each equation of the form:$A(x)y' + B(x)y + C(x) = 0$if we divide it with A(x) ≠0, becomes:$y' + P(x)y + Q(x) = 0$and is called linear differential equation of the first order.

3. 3. Bernoulli's differential equation

By generalization of a linear differential equation, Bernoulli's differential equation is gained of the form:$y' + P(x)y + Q(x)y^m = 0$where it is assumed that m ≠ 0 andm≠ 1, and form = 0 the above equation represents inhomogeneous linear differential equation, while for m = 1 it represents homogeneouslinear equation (Bernoulli's equation)$y' + P(x)y + Q(x)y = y' + [P(x) + Q(x)]y = y0 + P_1(x)y = 0$

3. 4. Equation with total differential

If there is differential equation:$P(x, y)dx + Q(x, y)dy = 0$andif the precondition: $\frac{\sigma P}{\sigma y} = \frac{\sigma Q}{\sigma x}$is fulfilled then it presents the equation with total differential. The left side of the equation presents the total differential of some function u(x, y)du = Pdx + Qdywhere: $P = \frac{\sigma u}{\sigma x}$ a $Q = \frac{\sigma u}{\sigma y}$

3. 5. Differential equations of higher order

The general form of the ordinary differential of n-th order is:ϕ (x; y'; y''; y''' : : : ; $y^{(n)}$) = 0i.e. if we solve the equation by $y^{(n)}y^{(n)}$ = f(x; y'; y''; y''' : : : ; $y^{(n-1)}$)

If the functionφ(x), which has continuous derivatives up to the n-th order, together with its derivatives satisfies the differential equation, that is, if the equation becomes an identity when y and derivatives of y substitute with φ(x) and derivatives ofφ (x) then the function φ (x) is called the solution to the differential equation.

With the use of Protégé editor, the open source platform, we will update only the division of differential equations and present them in Figure 6.

4. Conclusion

In this paper we presented a new concept of updating and using mathematical knowledge based on knowledge bases. Appropriate software and new technological solutions enable fast, efficient and inexpensive access to necessary mathematical knowledge in solving mathematical equations. It was not our goal to update all the necessary knowledge but only to propose a new concept for quick access to it.

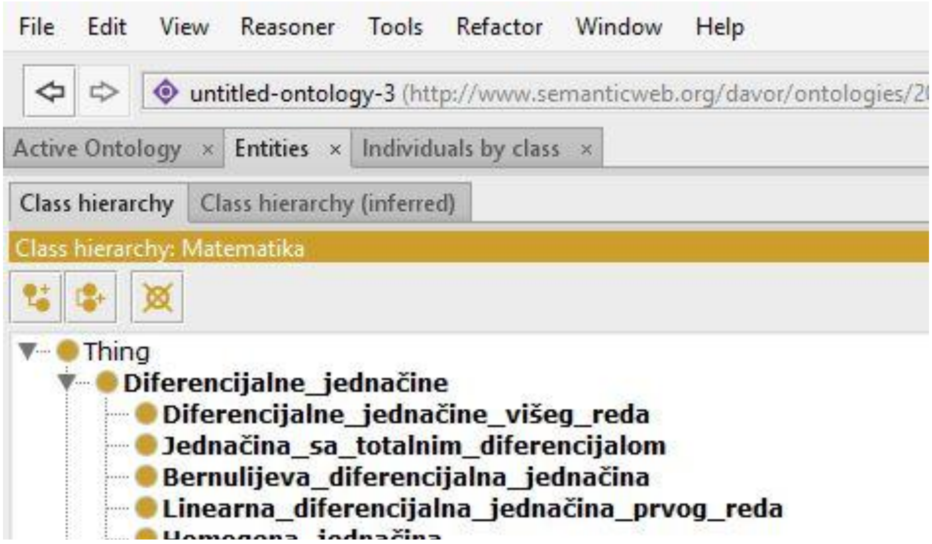

Figure 6. Differential equations

We had a problem because the used editor does not allow the entry of symbols in the database but onlythe text. Therefore, when updating mathematical knowledge, one should keep this fact in mind. For future updating of mathematical knowledge should be used some other software that has all the necessary features.

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